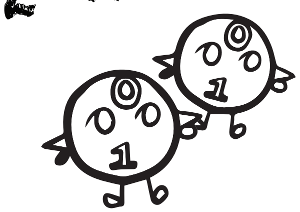
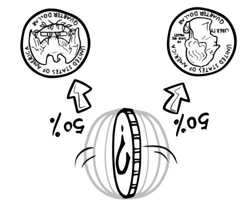
2 QUBITS



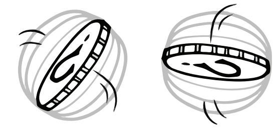
NOTATION & OPERATIONS

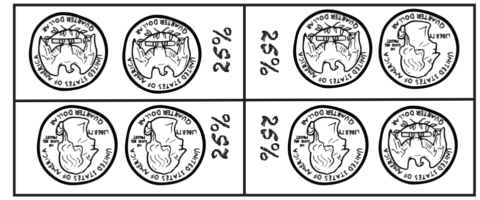
Two coin flips are independent

One coin:



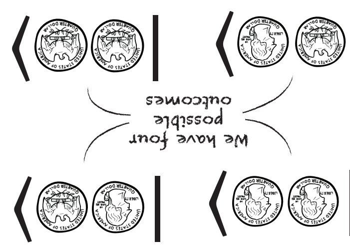
Two coins:





Dirac Notation

(Bra-ket)



Recall, in Dirac notation, the probability of some outcome $a|\cdot\rangle$ is $|a|^2$

We can express this state as...

$$\frac{1}{2}|HH\rangle + \frac{1}{2}|HT\rangle + \frac{1}{2}|TH\rangle + \frac{1}{2}|TT\rangle$$

Same probability for each outcome

2-Qubit Notation

If we measure two qubits, how many possible outcomes are there?

1st qubit → $|0\rangle|0\rangle$ ← 2nd qubit

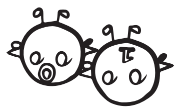
$|0\rangle|1\rangle$

$|1\rangle|0\rangle$

$|1\rangle|1\rangle$

As shorthand, we write:

$|1\rangle|0\rangle$ as $|10\rangle$



So if all measurement outcomes are equally likely, we have a state of...

$$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

Combining Two Qubits

Two independent (not entangled) qubits:

Qubit 1 $a|0\rangle + b|1\rangle$

Qubit 2 $c|0\rangle + d|1\rangle$

The same two qubits, expressed in 2-qubit notation:

$$ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

Try it yourself!

Put these qubits in 2-qubit notation:

Qubit x $\frac{1}{\sqrt{3}}|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}|1\rangle$

Qubit y $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

(Check your answer on the next page!)

Vector Notation

The 2-qubit state from the previous page can also be written as a vector!

$$\frac{1}{2\sqrt{3}}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$\begin{bmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Linear Algebra

Matrix multiplication is used to perform gate operations

C-NOT

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

C-NOT operation — input — output

Try it yourself!

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{7}} \\ \frac{\sqrt{3}}{\sqrt{7}} \\ \frac{\sqrt{2}}{\sqrt{7}} \\ \frac{1}{\sqrt{7}} \end{bmatrix} = ?$$

input

Answer: $\begin{bmatrix} \frac{\sqrt{3}}{\sqrt{7}} \\ \frac{1}{\sqrt{7}} \\ \frac{\sqrt{2}}{\sqrt{7}} \\ \frac{1}{\sqrt{7}} \end{bmatrix}$ output

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AN NSF EXPEDITION IN COMPUTING EPiQC

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