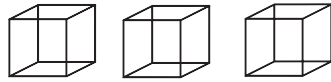


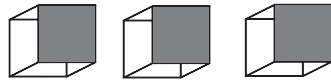
Quantum Entanglement



Your brain ENTANGLES visual data



Look at the top row of boxes. They all face the same way! Either up to the right or down to the left.

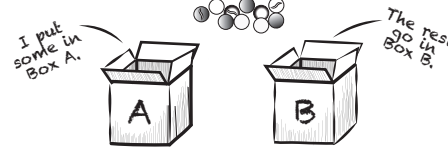


Without a reference your brain can see them both ways.

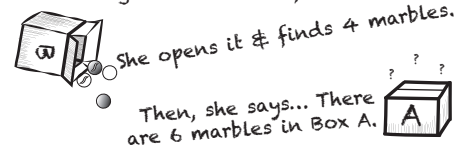
ENTANGLED with MATH

Sometimes TWO things are DEPENDENT

For example:
I have 10 marbles.



I close the boxes & give Box B to a friend



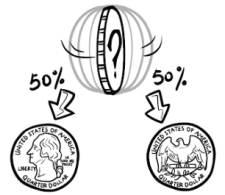
How did she know how many were in Box A WITHOUT opening it???

Because...

The number of marbles in each box is DEPENDENT on the number in the other box!

If coin flips are independent

One coin:



Two coins:



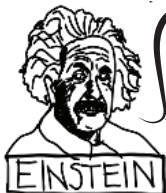
		25%			25%
		25%			25%
		25%			25%
		25%			25%

What if coin flips were entangled?!

We could create the following scenario:



		0%			50%
		0%			50%
		0%			50%
		0%			50%

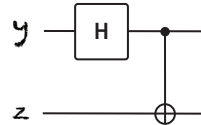


Inconceivable!
I don't believe it.
The math must be wrong!

ENTANGLEMENT in the QUANTUM world

Qubits become entangled in a special way.

Consider.... $y = |0\rangle$ and $z = |0\rangle$
in this quantum circuit:



There's a 50/50 probability of measuring them in the same state, but never in opposite states!

$$\frac{1}{\sqrt{2}}|00\rangle + 0|01\rangle + 0|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

For this circuit, when one of the entangled qubits is measured, the other is forced to take the same value.

Now the math Einstein didn't trust...

1. The Hadamard Gate acts on y , producing y'

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2. To apply a 2-qubit gate, first combine the probabilities for y' and z .

$$y' = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \text{ and } z = |0\rangle + 0|1\rangle$$

$$\frac{1}{\sqrt{2}}|00\rangle + 0|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

Convert to matrix notation $\rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

3. The C-NOT Gate acts on z and y'

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Quantum Gates are no longer a math game.

Physicists can really build them!

<https://www.epiqc.cs.uchicago.edu/resources/>

March 2019 (v2)

This work is funded in part by EPIQC, an NSF Expedition in Computing, under grant 1730449

