Exponential Growth

Explore how quickly numbers can grow

varies

8+ years

Learning Goals

Students will:

- Explain how exponential growth occurs.
- Understand how exponential growth is important to quantum computing.

Importance in Quantum Computing

The number of operations a quantum computer can perform grows exponentially because each qubit can occupy two states simultaneously.

Materials

Growth on a Checkerboard activity:

- ❏ Checkerboard
- ❏ 31 identical small objects (e.g, pennies)

Colored Tiles activity:

- ❏ 32 (or more) small tiles of the same size (e.g., dominoes, Magna-Tiles)

Paper Folding activity:

- ❏ 2 (or more) sheets of paper

Exponential Growth Exit Ticket (optional)

Preparation

Choose one (or more) of the activities, depending on the available materials and time. The more ways that participants can engage with the concept of exponential growth, the better they will understand it.

If you plan to use the *Colored Tiles* activity, identify a space to cover with the tiles (e.g., table top).

Background Knowledge

If it's been awhile since you've thought about *exponents*, remember the basis for this mathematical operation is multiplication. If we raise a number (n) to the power of 2 (n^2), we multiply that number by itself ($8^2 = 8 \times 8 = 64$). A common mistake is to multiply the number by the exponent ($8 \times 2 = 16$).

On an exponential scale, the rate of change is *ever increasing*. Think about raising the number 2 to consecutive exponents. 2 is a small number to start with, but as we raise it to higher exponents: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$… Exponents are a powerful mathematical tool with many applications.

In quantum computing, qubits function on an exponential scale. In classical computing, each bit occupies a single state at a given time. In quantum, however, qubits have the ability to be in superposition, which means they can occupy two states simultaneously. Therefore, when you add a qubit to a quantum computer, the number of possible states of the system doubles.

Facilitating the Activity

ENGAGE

Literacy Connection

1. Consider reading or having available one or more of the following:
    a. *On Beyond a Million: An Amazing Math Journey* by David M. Schwartz
    b. *One Grain of Rice: A Mathematical Folktale* by Demi

In these books, the characters experience numbers that grow exponentially. Ask questions about exponential growth: *What do you notice about the numbers in the story? Have you ever seen patterns like this before? What do you think the next number might be?*

ACTIVITIES

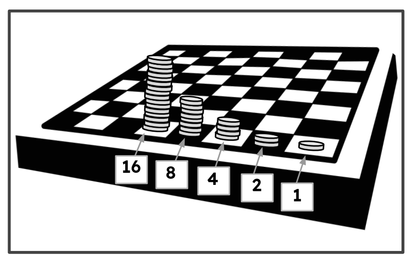
Facilitation Note: Choose one or more of the following activities, depending on the available materials and time. The more ways that participants can engage with the concept of exponential growth, the better they will understand it.

<u>Growth on a Checkerboard</u>

1. Start with a blank grid. (A standard sized chess/checkers board will have 64 squares to work with, but you can use a grid of any size - the process will be the same).

2. Tell participants that you are going to place one object in the first square and continue to fill the squares of the grid by putting twice the number of objects in a square as there are in the previous square (e.g., the second will have two objects).

a. Ask participants: How many objects will be in the next square? [*There will be twice as many as in the previous square, or 2 x 2 = 4 objects.*]



3. Place 1 object in the first square, then 2, then 4, then 8, and so on. If you start with 31 items, you have enough items to fill 5 squares: 1 + 2 + 4 + 8 + 16 = 31.

4. Ask participants:
    How many objects do you think we will need in the last square? *[For a standard chess/checkers board with 64 squares, you would need 2^{63} objects, which is **9,223,372,036,854,775,808** or 9 quintillion 223 quadrillion 372 trillion 36 billion 854 million 775 thousand 808.]*

    Do you know a name for this kind of growth? [*This is called **exponential growth**.*]

    Do you know any examples of exponential growth? [*On a global scale, the human population is growing exponentially, doubling in size in about 50 years. Bacteria often show exponential growth.*]

Colored Tiles

Facilitation Note: This activity is similar to Growth on a Checkerboard, but instead of adding double the amount of small objects to each new square on the board, we are laying tiles side by side all in one space.

1. Hold up one tile, and ask participants: How many tiles do you think could fit in this space (e.g., a piece of paper, tabletop)? If we place 1 tile, then 2 more, and then four…so we are doubling the number of tiles each time…how quickly do you think we would fill the space?

2. Work with participants to place tiles in your space. Place 1 tile, then place 2 more adjacent to the first.

   Ask: We added twice as many tiles as were there before. Did it cover as much space as you thought it would? Can you imagine what it would look like to add four more tiles?

3. Work with participants to place 4 more tiles (7 total). Then, continue the pattern until you run out of either tiles or space. Participants will get a sense of how quickly the tiles are covering the space. Tell participants that when you double the number each time, that's called an ***exponential pattern***.

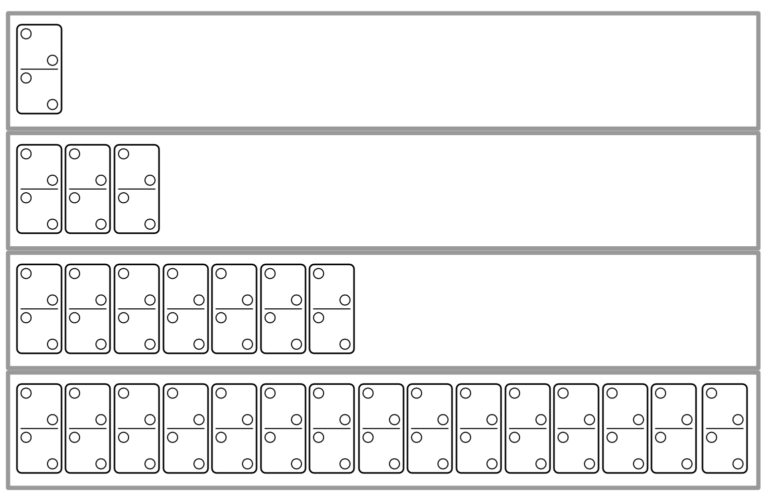
4. Work with participants to figure out how many times you added tiles to the space. For example, a space covered with 15 tiles, required 4 iterations: 1 tile, add 2, add 4, add 8 because 15 = 1 + 2 + 4 + 8.)

First iteration:
1 tile

Second iteration:
3 tiles

Third iteration:
7 tiles

Fourth iteration:
15 tiles



Paper Folding

1. Hold up a piece of paper, and ask participants like the following:
   - How many times (in a row) do you think you can fold a piece of paper?
   - Have you ever tried to fold a piece of paper many times in a row? If so, what happened?
   - How many sections do you get when you fold a piece of paper in half and then unfold it?
   - How do you think the number of sections relates to the number of folds?
   - Each time you fold the paper, how many sections are added?
2. Have a participant fold the paper once and unfold it. Ask what they observe. [*There are two sections, separated by the fold. See Figure 1.*]
3. Ask participants to predict what will happen if the paper is refolded, and then folded in half a second time.
4. Have a participant refold the paper and then add a second fold. How many sections are there when you unfold it? [*There are 4 - or twice as many*.]

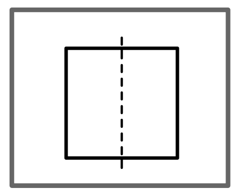


Figure 1

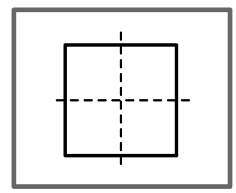


Figure 2

5. Ask participants to predict:
   What will happen after a 3rd fold is added. [*There will be 8 sections*]
   How many will be there when a 4th fold is added? [*There will be 16 sections*].
6. Have participants add a third and fourth fold and check their predictions.

   Ask: Do you notice a pattern? [*The number of sections doubles each time you add a fold. The number of sections* ***grows exponentially.***]
7. Ask participants:
   What have you noticed about the thickness of the paper as you fold it? Has it changed? [*The paper gets thicker each time you fold it.*]

   How many times do you think we can fold the paper before we can't fold it any more? Have a participant see how many times they can fold it. [*In theory, you can fold a standard piece of paper no more than 7 times, which would give you 128 sections.*]
8. Tell participants that just like the number of sections grows exponentially with each fold, so does the thickness of the paper as the layers stack up. Have participants observe an unfolded piece of paper and piece of paper participants folded as many times as they could to reinforce the difference in thickness between the two.

9. Ask participants to predict: How tall do you think the paper would be if we were able to fold it 17 times? Wait for participants to make predictions, even if they seem like wild guesses. Then, tell them that they can figure it out using exponents!
    a. If the paper is folded 17 times, the number of sections would be 2^{17} = 131,072.
    b. Assuming the unfolded paper is 0.001 cm thick, the folded paper would be 0.001 cm x 131,072 = 131.072 cm or 51.6 inches tall.
10. Ask participants to make similar predictions for other numbers of folds.
    a. 25 folds:

        <u>Number of sections</u>: 2^{25} = 33,554,432 sections
        <u>Height</u>: 33,554,432 × 0.001 cm = 33,554.432 cm / 13,210.4 inches / 1,100.9 ft

        This is about the same height as the Chrysler Building in New York City (a little less than a quarter of a mile).

    b. 30 folds:

        <u>Number of sections</u>: 2^{30} = 1,073,741,824 sections
        <u>Height</u>: 1,073,741,824 x 0.001 cm = 1,073,741.824 cm / 422,2733 inches / 35,227.8 ft / 6.7 miles.

        Commercial airplanes fly at an altitude of about 35,000 feet!

    c. 40 folds:

        <u>Number of sections</u>: 2^{40} = 1,099,511,627,776 sections
        <u>Height</u>: 1,099,511,627,780 x 0.001 cm = 1,099,511,627.78 cm / 432,878,594 inches / 36,073,216.1 ft / 6,832 miles.

        That's about the distance from the United States to China!

    d. 45 folds would make a paper stack that could reach the Moon!

DISCUSSION

Quantum Connection

Tell participants that researchers are developing a new kind of computer called a quantum computer, which uses the power of exponents! Quantum computers are not simply faster than classical computers. Rather, they are able to solve problems in very different ways than classical computers can. This is because of the exponential growth aspects of the qubits used in quantum computers and their superposition abilities. When one qubit is added to a quantum computer, the number of possible states of the system doubles. For certain types of problems, quantum computers can pick out patterns and come to a solution much more quickly, because they can look at all possible combinations at once.

Tell participants that quantum computers are well-suited for problems that require looking at **many** possible solutions. Ask: Can you think of any really big problems that quantum computers might be useful for?

Ask participants to think back to the exponential growth activities. We ended up with immensely huge numbers! Now, apply the concept of exponential growth to building a quantum computer. Each time a qubit is added to the computer, the number of operations the computer can perform doubles:

> A quantum computer with 64 qubits could perform 2^{64} = 18,446,744,073,709,551,616 operations simultaneously in order to answer a question!
>
> A classical computer would take about 400 years to complete the same problem, because it would have to run through each of the 18,446,744,073,709,551,616 possible answers one by one.

Here are some examples of problems that can only be solved once we have more powerful quantum computers:

1. **Designing new medicines**: Scientists are interested in finding new medicines and many chemical reactions are still not completely understood. Quantum computers could replicate chemical systems to give us new insights into molecules and reactions by simulating how the electrons in the atoms that make up molecules interact with each other.
2. **Cracking codes and better encryption**: Our daily lives rely on modern cryptography, in everything from banking to secure web searches. Computer cryptography is based on the fact that it is much easier to multiply two numbers to find their product than it is to find specific factors of a given number — and this gets harder as the numbers get larger. Current encryption algorithms, called RSA

protocols, can be broken if one can figure out the two prime factors of a number with hundreds of digits. This is a problem that classical computers would need an enormous amount of time to solve. However, an algorithm on a quantum computer could quickly calculate the prime numbers used in current encryption schemes. Currently, quantum computers are too small and error prone to accomplish this. However, once quantum computers get large enough, new encryption schemes will need to be developed that are unsolvable by quantum computers.

3. **Designing new fertilizers**: The Haber Process is the basis for creating fertilizer, which is key in food production. Scientists hope quantum computers will give them a better understanding of this process in the near future. This would help scientists find more energy-efficient ways to make fertilizer which in turn increases food production. Solving this problem requires a quantum computer with between 1,000 and 10,000 qubits, but the biggest quantum computer right now is 100 qubits.

4. **Finding meaning in Big Data**: Many compelling questions in science boil down to finding patterns in large datasets. Finding these patterns is harder as the datasets get larger - and they are getting huge in many scientific fields. Quantum computers offer a fundamentally different and faster way to explore these large datasets and could help solve this important type of problem. On a practical level, quantum computers could make web searches faster and better.

5. **Designing better solar energy materials**: Quantum computers could help scientists and engineers design better photovoltaics, which are solar energy cells that absorb sunlight and convert it to electricity. Solar energy is an important type of renewable energy and quantum computers could help us find better, more efficient ways to make them.

Connections to Standards

Next Generation Science Standards*

Crosscutting Concepts: Scale, Proportion, and Quantity
Science and Engineering Practices: Using Mathematical and Computational Thinking

Common Core State Standards

Standards for Mathematical Practice: Reason Abstractly and Quantitatively, Model with Mathematics, Use Appropriate Tools Strategically

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Exponential Growth

Exit Ticket

<u>Directions</u>: Read the passage and answer the questions.

<u>MAYA'S ROCK COLLECTION</u>

Nia and her little sister, Maya, share a small bedroom. Maya has a large rock collection on display in their room.

Nia recently learned about exponential growth at school, and she realized that the number of rocks in Maya's collection is growing exponentially!

1. If Maya's rock collection is *growing exponentially*, that means the total number of rocks in her collection is ____________ .

    a. Slowly decreasing
    b. Slowly increasing
    c. Rapidly decreasing
    d. Rapidly increasing
    e. Not changing

2. Select the scenario(s) that describes Maya's rock collection as growing exponentially:

    - ☐ <u>Scenario A</u>: Nia and Maya go for long walks with their family each weekend. Maya often stops to look at rocks. She picks out her two favorite rocks and brings them home to add to her collection.

    - ☐ <u>Scenario B</u>: Nia and Maya look forward to their uncle visiting each weekend. He always brings a book for Nia and rocks for Maya. The first time, he brought one rock for Maya. The next week, he brought two rocks. Each week since then, he has brought twice as many rocks as the week before.

3. I think this because ____

    ____

    ____

ANSWER KEY

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3. I think this because... Maya's rock collection grows very rapidly in Scenario B, because it increases by a number that doubles each week (1, 2, 4, 8, 16, 32, 64, 128 rocks), whereas the growth described in Scenario A is much slower, increasing by the same amount each week (2 rocks per week).