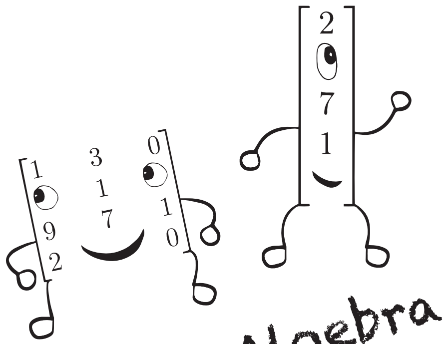
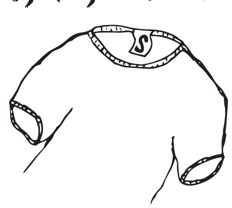


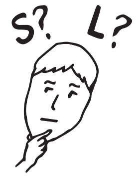
Linear Algebra

Just the basics

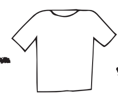
T-shirt labels often say S, M, or L



In isolation, S, M, or L could mean almost anything



But in the context of a t-shirt, they represent the minimum amount of information needed to determine the size

S +  = "small"

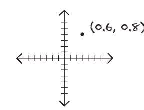
Vectors

A vector is just an ordered lists of numbers

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \quad \begin{bmatrix}\pi\\0\end{bmatrix} \quad \begin{bmatrix}2019\\6\\28\end{bmatrix}$$

But without any context, a vector such as $\begin{bmatrix}0.6\\0.8\end{bmatrix}$ could be anything!

Coordinates?



A Polynomial?

$f(x) = 0.6x + 0.8$

A quantum state?

$0.6\,|0\rangle + 0.8\,|1\rangle$

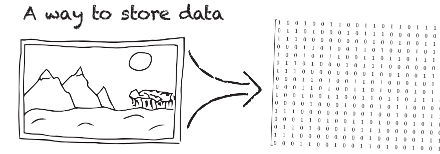
Matrices

A matrix is just a 2-dimensional ordered collection of numbers

$$\begin{bmatrix}1 & 2 & 0\\0 & 1 & 0\end{bmatrix} \quad \begin{bmatrix}2 & 4\\-3 & 8\end{bmatrix}$$

And like a vector, a matrix can mean a lot of different things

A way to store data



A matrix can also represent an operator that transforms vectors

$$\underset{\text{initial vector}}{\begin{bmatrix}1\\0\end{bmatrix}} \rightarrow \overset{\text{operator}}{\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}} \rightarrow \underset{\text{resulting vector}}{\begin{bmatrix}0\\-2\end{bmatrix}}$$

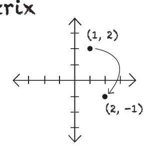
Matrix Multiplication

We can use matrices to transform vectors into different vectors, by multiplying a matrix and a vector

$$\underset{\text{operator}}{\begin{bmatrix}a & b\\c & d\end{bmatrix}} \overset{\text{multiplication}}{\times} \underset{\text{initial vector}}{\begin{bmatrix}x\\y\end{bmatrix}} = \underset{\text{new, transformed vector}}{\begin{bmatrix}ax+by\\cx+dy\end{bmatrix}}$$

For example, rotation 90° can be represented by the matrix

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$



Rotating the point (1,2):

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\times 1+1\times 2\\-1\times 1+0\times 2\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix}$$

(We often omit the multiplication sign)

Quantum Gates

Qubits can be written as vectors. Quantum gates transform qubits.

$$-\boxed{X}- \rightsquigarrow \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

So we can write gates as matrices.

Let's see what this gate does when we give it the qubit $0.6\,|0\rangle + 0.8\,|1\rangle$

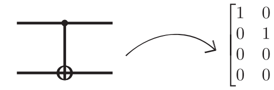
$$\begin{aligned}\underset{-\boxed{X}-}{\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}}\begin{bmatrix}0.6\\0.8\end{bmatrix} &= \begin{bmatrix}0\times 0.6+1\times 0.8\\1\times 0.6+0\times 0.8\end{bmatrix}\\ &= \begin{bmatrix}0.8\\0.6\end{bmatrix}\\ &= 0.8\,|0\rangle + 0.6\,|1\rangle\end{aligned}$$

Bigger Matrices

We use bigger matrices to transform bigger vectors.

$$\begin{bmatrix}a_{1,1} & a_{1,2} & \cdots & a_{1,n}\\a_{2,1} & a_{2,2} & \cdots & a_{2,n}\\\vdots & \vdots & \ddots & \vdots\\a_{n,1} & a_{n,2} & \cdots & a_{n,n}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix} = \begin{bmatrix}a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n\\a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n\\\vdots\\a_{n,1}x_1 + a_{n,2}x_2 + \cdots + a_{n,n}x_n\end{bmatrix}$$

The CNOT gate is a quantum gate that operates on 2 qubits, so we use a 4x4 matrix to represent it.



$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$

Try out the example below of a CNOT gate acting on a two qubit state!

Compute:

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

Answer: $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

Find more Quantum Computing zines here:

https://www.epiqc.cs.uchicago.edu/resources/

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