30 min

6+ years

Measurement Perturbs State

Explore how measuring something can change it

Learning Goals

- Understand that measurement methods can change the thing being measured

Importance in Quantum Computing

The state of quantum bits, or qubits, changes when you measure it.

Materials

- ❏ Balloons, each labeled with a number
- ❏ Substances for filling balloons, such as: flour, rice, cornstarch, dried beans, pennies, sand, salt, corn syrup, dish soap, water
- ❏ Funnel
- ❏ Straw
- ❏ Bowls, each labeled with a number
- ❏ *Measurement* worksheet

Preparation

- Fill each balloon with approximately ¼ cup of a substance. For solids and liquids, place the straw and stem the funnel in the opening of the balloon. Spoon or pour the substance into the funnel to fill the balloon. Tie the balloons closed.
- For an air-filled balloon, blow up one balloon to approximately the same size as the others and tie it closed.
- Print the *Measurement* worksheet.

Background Knowledge

Usually when something is measured, it is the same before and after we measure it. If you measure the length of a book using a ruler, the book is the same before and after measurement. Sometimes, however, measurement can change the thing we are measuring. For example, we might want to know how many licks it takes to get to the center of a Tootsie Pop; licking the Tootsie Pop until you get to the center means the Tootsie Pop changes as you measure it. In science, it is often important that the method of measurement does not perturb, or change, what you are measuring. Classical computers store simple values in memory, and the storage devices are able to both measure and retain the value when you read out of memory. Quantum computers, however, have a very complex, fragile state at the molecular level. No measurement device exists that can measure qubits without fundamentally changing what is being stored.

Facilitating the Activity

ENGAGE

Literacy Connection

1. Consider reading or having available one or more of the following:
    a. *Inch by Inch* by Leo Lionni
    b. *The Three Little Pigs* (choose your favorite version)
    c. *Goldilocks and The Three Bears* (choose your favorite version)

   In these stories, characters measure various items (e.g., the length of a bird's tail in *Inch by Inch*, the strength of buildings in *Three Little Pigs*, and the temperature of porridge in *Goldilocks and The Three Bears*). Ask participants questions to get them thinking about how we measure different things: What were they measuring? How did they measure it? Why did they measure it that way? Did it change at all when they measured it?

ACTIVITY

1. Show participants the balloons you have prepared. Tell participants that you need their help to figure out what is in each balloon, but that the balloons can't be opened. Ask: What ideas do you have about how we could figure out what is in the balloons? What measurements could we collect to help us figure it out?
2. As participants share their ideas, categorize the responses and ask leading open-ended questions such as:
    a. What would you use to help you decide? What is your measurement device? (e.g., your eyes, your ears)
    b. What would you do to figure it out? What is your measurement process?
    c. What do you think will happen when you do that?
    d. How will that help you determine what is in the balloons?
3. If it hasn't come up already, suggest using our sense of touch.
4. Have participants touch the different balloons and ask them to describe the properties they notice.
5. Ask questions such as:
    a. What do you think is in each balloon?
    b. How do your observations help you figure out what's in each balloon?
    c. Which balloons do you think have solids in them? Why do you think that? What about liquids? Gases?
    d. What changed during this investigation?

Facilitation Note: Consider recording participants' observations and predictions of the contents of each balloon on chart paper, a whiteboard, etc.

6. If it hasn't come up, tell participants that feeling the balloons did not change, or <u>perturb</u>, the balloons. But it also did not allow us to know for sure what they contained. For example, maybe you figured out that a balloon contains marbles, but you can't determine their color. Ask participants for their ideas on how we could find out for sure what's in each balloon.
7. If it doesn't come up, suggest cutting open the balloons. Note that this will change, or perturb, the balloons but will also let us know what is in each.
8. Cut open the balloons and empty their contents into the corresponding bowl. Compare participants' predictions to the results. Ask questions such as:
    a. Were you correct?
    b. Which substances were easiest to identify? Hardest?
    c. Why do you think that is?

DISCUSSION

1. Facilitate a discussion of the idea that measuring something about an object can change it. Ask questions such as:
    a. How did with measure the contents of the balloons without changing them? What did we do that did change them?
    b. Can you think of any other times when we measure something and it changes what we are measuring? [*Some examples include testing foods to see if they are done cooking (you eat some of the food), checking the air pressure of tires (some air escapes), and conducting car "crash tests" (the car is damaged or destroyed).*]
    c. Do you think it's important to know if the measurement changes the object?

Quantum Connection

2. Tell participants that in quantum computers, the qubits can be in two states at once. When we try to measure the state of a qubit, it changes the qubit so that it is forced into one of the two possible states. On the quantum level, current measurement methods interfere with the values we are attempting to measure. As soon we try to measure something on the quantum level, the very event/entity we're trying to measure changes.
3. Ask participants to complete the *Measurement* worksheet. Consider facilitating a discussion so that participants can share their thoughts, once they have finished.

Connections to Standards

Next Generation Science Standards*

Crosscutting Concepts: Cause and Effect, Stability and Change
Science and Engineering Practices: Planning and Carrying Out Investigations, Using Mathematics and Computational Thinking

Common Core State Standards

Standards for Mathematical Practice: Construct Viable Arguments and Critique the Reasoning of Others, Use Appropriate Tools Strategically

Acknowledgements

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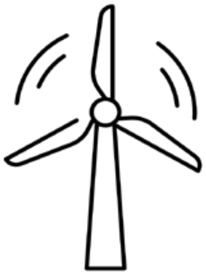
I am in grade __________

Measurement

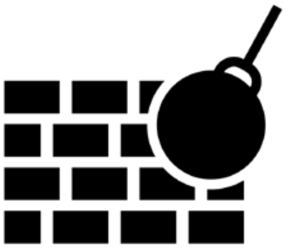


Nevaeh's class is learning about windmills. She and her partner designed blades for a windmill and then attached them to the windmill. They will test their blades by measuring how fast the windmill turns in the wind. They will record how many times the blades turn in 1 minute. Nevaeh will use her eyes to count the number of turns and her partner will use a stopwatch to time 1 minute.

Will this measurement change the state of the windmill blades?
Explain your thinking.

I am in grade __________

Measurement

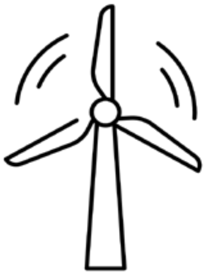


Marisol's class has been learning about how walls are built. They designed a mortar made out of sand, clay, and water to hold the bricks together. Now they will test the strength of their mortar by hitting the wall with a wrecking ball. They will use their eyes to look at the wall after the wrecking ball hits it. They will count how many times the wrecking ball hits the wall before it falls down.

Will this measurement change the state of the mortar?
Explain your thinking.

I am in grade __________

Measurement



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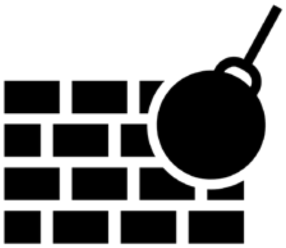
Will this measurement change the state of the windmill blades?
Explain your thinking.

Answers will vary, but look for thinking related to how the windmill blades (or the windmill as a whole) should stay the same during the measurement.

I am in grade __________

Measurement



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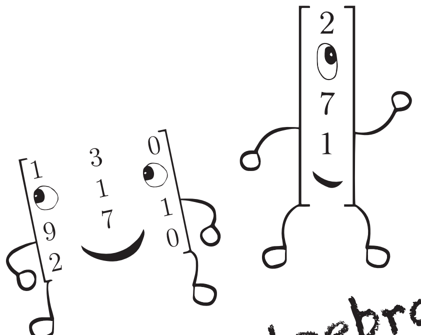
Will this measurement change the state of the mortar?
Explain your thinking.

Answers will vary, but look for thinking related to how the mortar (or the wall as a whole) is likely to change because it will break when the wall falls down.

Share what you've learned about how measuring something can change it!

- Explain to someone else what it means for a measurement to change the thing that you are measuring.

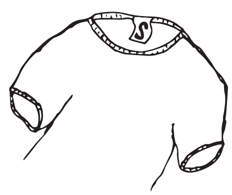
- Notice when you change things by measuring them in your everyday life. Discuss your experiences with others.
    - What were you measuring?
    - How were you measuring it?
    - What changed when you measured it?



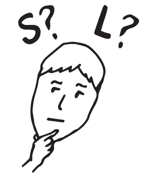
Linear Algebra

Just the basics

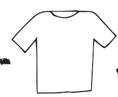
T-shirt labels often say S, M, or L



In isolation, S, M, or L could mean almost anything



But in the context of a t-shirt, they represent the minimum amount of information needed to determine the size

S +  = "small"

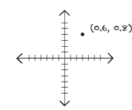
Vectors

Vectors are just ordered lists of numbers

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \quad \begin{bmatrix}\pi\\0\end{bmatrix} \quad \begin{bmatrix}2019\\6\\28\end{bmatrix}$$

But without any context, a vector such as $\begin{bmatrix}0.6\\0.8\end{bmatrix}$ could be anything!

Coordinates?



A Polynomial?

$f(x) = 0.6x + 0.8$

A quantum state?

$0.6\,|0\rangle + 0.8\,|1\rangle$

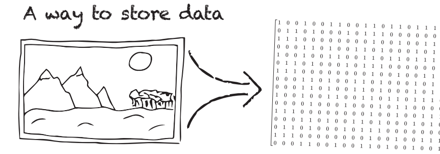
Matrices

A matrix is just a 2-dimensional ordered collection of numbers

$$\begin{bmatrix}1 & 2 & 0\\0 & 1 & 0\end{bmatrix} \quad \begin{bmatrix}2 & 4\\-3 & 8\end{bmatrix}$$

And like a vector, a matrix can mean a lot of different things

A way to store data



Transforming vectors

$$\underset{\text{initial vector}}{\begin{bmatrix}1\\0\end{bmatrix}} \rightarrow \overset{\text{operator}}{\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}} \rightarrow \underset{\text{resulting vector}}{\begin{bmatrix}0\\-2\end{bmatrix}}$$

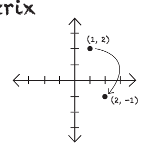
Matrix Multiplication

We can use matrices to transform cevtors into different vectors, by multiplying a matrix and a vector

$$\underset{\text{operator}}{\begin{bmatrix}a & b\\c & d\end{bmatrix}} \overset{\text{multiplication}}{\times} \underset{\text{initial vector}}{\begin{bmatrix}x\\y\end{bmatrix}} = \underset{\text{new, transformed vector}}{\begin{bmatrix}ax+by\\cx+dy\end{bmatrix}}$$

For example, rotation 90° can be represented by the matrix

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$



Rotating the point (1,2):

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\times1+1\times2\\-1\times1+0\times2\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix}$$

(We often omit the multiplication sign)

Quantum Gates

Qubits can be written as vectors. Quantum gates transform qubits.

$$-\boxed{X}- \rightsquigarrow \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

So we can write gates as matrices.

Let's see what this gate does when we give it the qubit $0.6\,|0\rangle + 0.8\,|1\rangle$

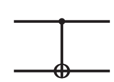
$$\underset{-\boxed{X}-}{\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}}\begin{bmatrix}0.6\\0.8\end{bmatrix} = \begin{bmatrix}0\times0.6+1\times0.8\\1\times0.6+0\times0.8\end{bmatrix}$$
$$= \begin{bmatrix}0.8\\0.6\end{bmatrix}$$
$$= 0.8\,|0\rangle + 0.6\,|1\rangle$$

Bigger Matrices

We use bigger matrices to transform bigger vectors.

$$\begin{bmatrix}a_{1,1} & a_{1,2} & \cdots & a_{1,n}\\a_{2,1} & a_{2,2} & \cdots & a_{2,n}\\\vdots & \vdots & \ddots & \vdots\\a_{n,1} & a_{n,2} & \cdots & a_{n,n}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix} = \begin{bmatrix}a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n\\a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n\\\vdots\\a_{n,1}x_1 + a_{n,2}x_2 + \cdots + a_{n,n}x_n\end{bmatrix}$$

The CNOT gate is a quantum gate that operates on 2 qubits, so we use a 4x4 matrix to represent it.



$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$

Try out the example below of a CNOT gate acting on a two qubit state!

Compute:

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

Answer: $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$