IMPROVED QUANTUM CIRCUITS VIA DIRTY QUTRITS

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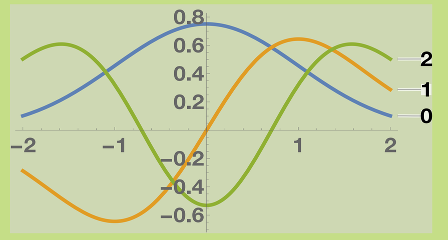
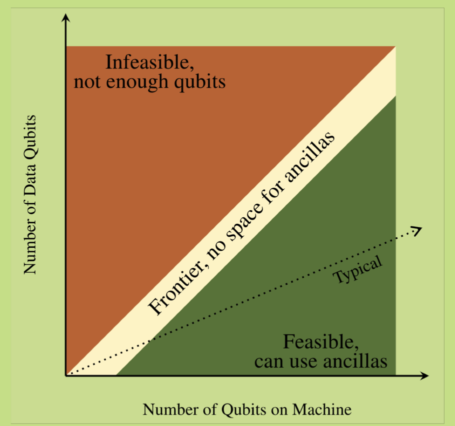
ABSTRACT

Current efforts towards scaling quantum computer focus on increasing # of qubits or reducing noise. We propose an alternative strategy:



Qutrits
Quantum systems have natural access to an infinite spectrum of discrete states→in fact, two-level qu*bits* require suppressing higher states. The underlying physics of qu*trits* are similar as those for qu*bits*.



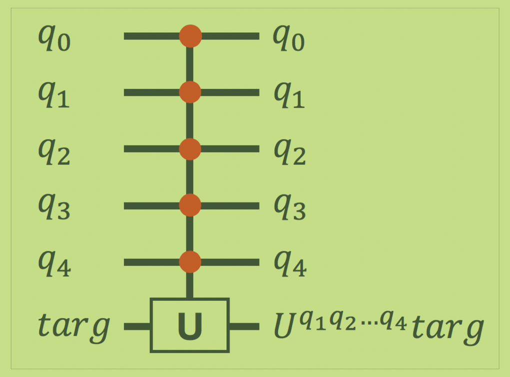
Our strategy replaces ancilla bits with qutrits. This enables operation at the ancilla-free frontier, where each hardware qubit is a data qubit.

In prior work, qutrits conferred only a lg(3) constant factor advantage via binary to ternary compression [1-4]. We introduce a new technique that maintains binary input and output, but uses intermediate qutrit states that replace ancillas. We show that this technique leads to asymptotically better circuit depths. Our circuit constructions have **polylog depth**, compared to linear depth for qubit-only circuits. We explore the tradeoff of operating higher-error qutrits by performing simulation under realistic noise models. Our simulations suggest that our techniques would significantly improve circuit fidelity by orders of magnitude, even for current device errors and sizes.

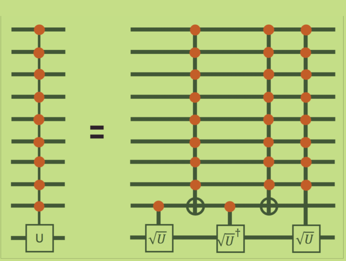
Our circuit constructions are applicable to a broad range of quantum algorithms, including Grover Search and Shor's Factoring [5, 6].

BACKGROUND

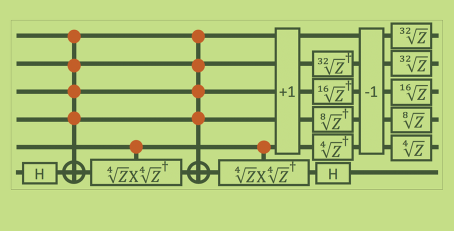
A critical operation for quantum computing is the multiple-controlled Toffoli, which applies an flip gate to a target iff all controls are $|1\rangle$



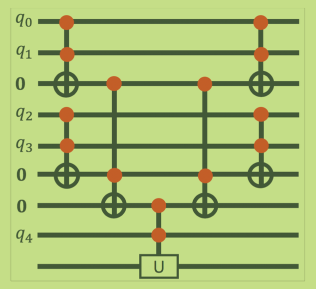
Prior Decompositions:



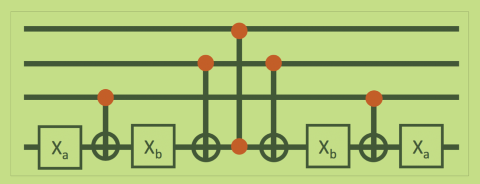
~N^2 depth
0 ancilla [7]



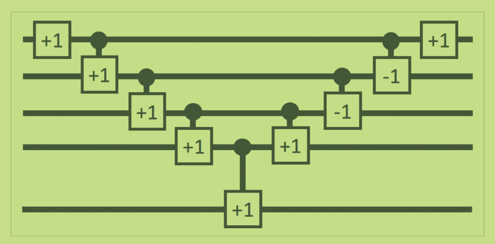
~N depth
0 ancilla [8]



~lg(N) depth
~N ancilla [9]



~N depth
N-level target [10]



~N depth
Qutrits [11]

CIRCUITS

Circuits represent quantum programs as a "timeline" of gates. All circuits have been verified.

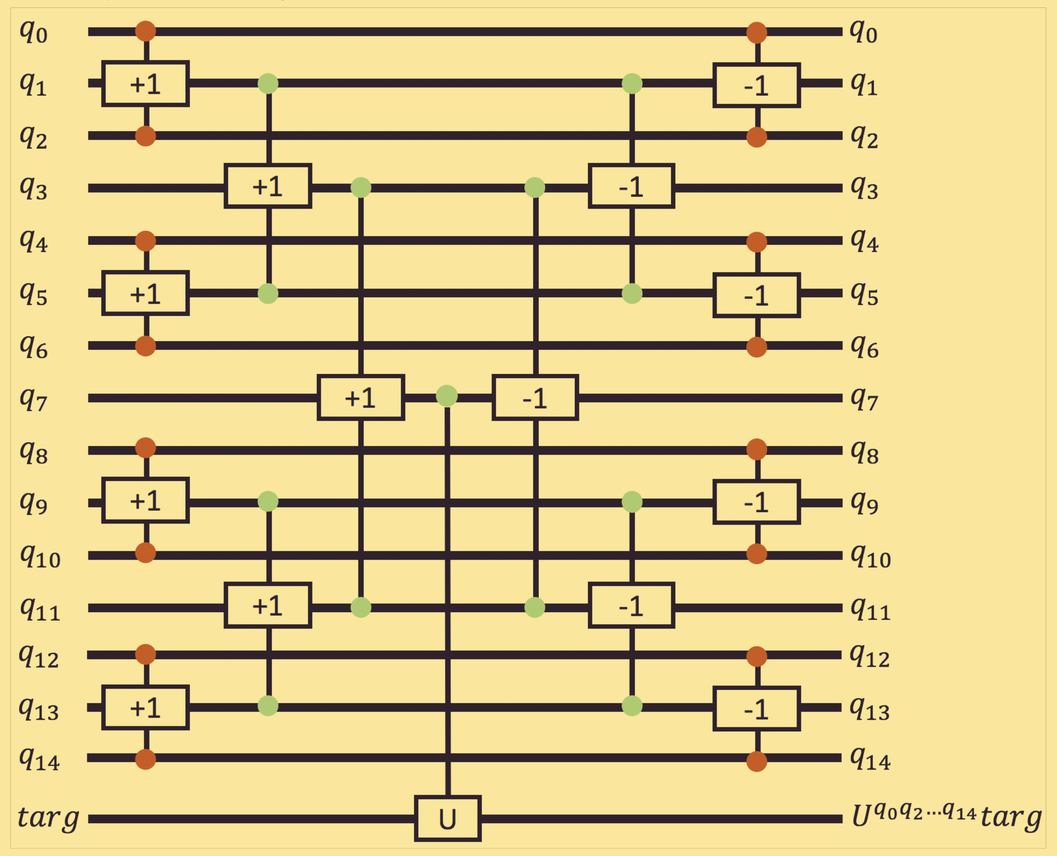
● = control on $|1\rangle$ ● = control on $|2\rangle$

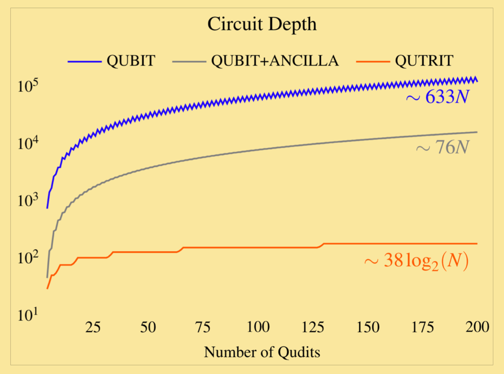
● = control on $|2\rangle$. If k = 1, also control on $|0\rangle$

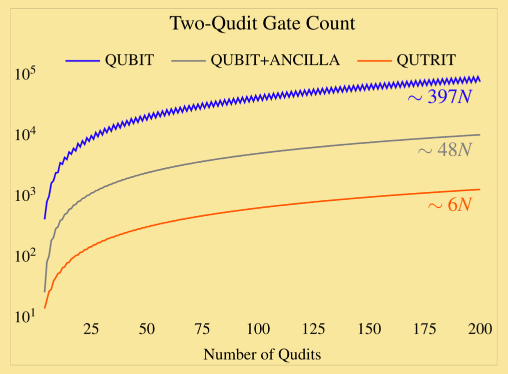
$+1|x\rangle = |x + 1 \bmod 3\rangle$ $\mathrm{FLIP}_{i,j}|i/j\rangle = |j/i\rangle$

Multi-Control

Apply U to target iff all control bits are $|1\rangle$

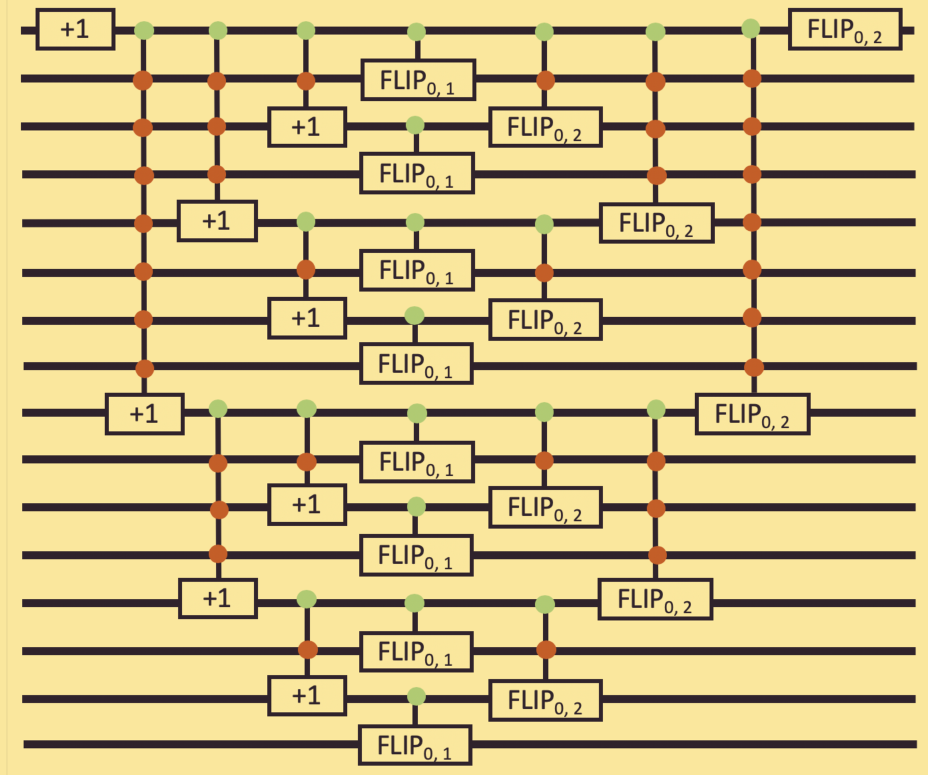






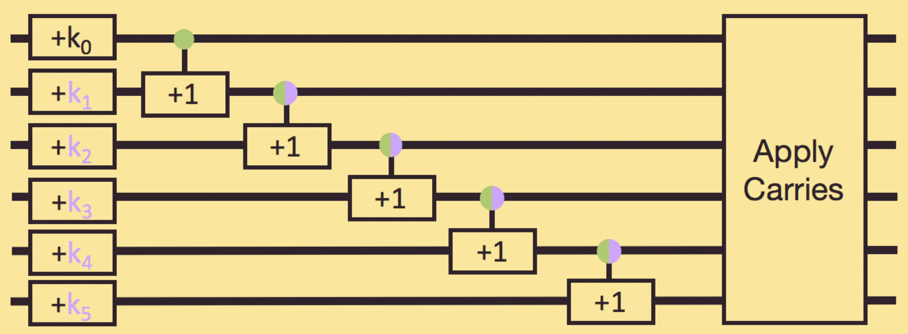
+1 (Incrementer)

Increment the input qubits (mod 2^N)



+k (Constant Adder)

Increment the input qubits by k (mod 2^N)



Other Applications

- Grover search (diffusion operator)
- Shor's algorithm (modular exponentiation)
- Quantum artificial neurons (multi-control Z)
- Logical qubit encoding for error correction

QUTRIT ERROR MODELING

We modeled depolarizing gate errors and amplitude damping idle errors for both superconducting and trapped ion systems.

Depolarizing [12]

For *qubits*, the singleton gate error channels are Pauli:

$$\sigma' = \mathcal{E}(\sigma) = p_1 X\sigma X + p_1 ZX\sigma XZ + p_1 Z\sigma Z + (1-3p)\sigma$$

$$= (1-3p_1)\sigma + \sum_{(j,k)\in\{0,1\}^2\setminus(0,0)}^{1} p_1 \,(X^j Z^k)\sigma(X^j Z^k)^\dagger$$

For *qutrits*, we extend to generalized Pauli operators:

$$\sigma' = \mathcal{E}(\sigma)$$

$$= (1-8p_1)\sigma + \sum_{(j,k)\in\{0,1,2\}^2\setminus(0,0)}^{1} p_1 \,(X_{+1}^j Z_3^k)\sigma(X_{+1}^j Z_3^k)^\dagger$$

We model similarly for two-qudit operations. The dominant effect is that errors increase from $8p_2$ to $80p_2$.

Amplitude Damping [13]

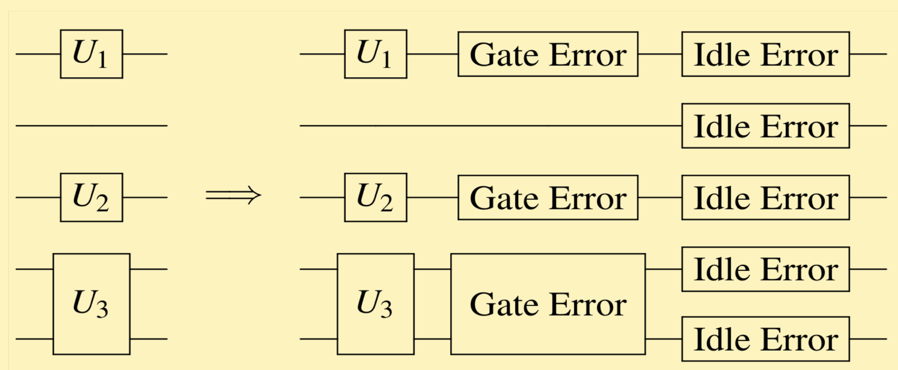
The amplitude damping errors for qutrits occur as idle errors with the following Kraus operators:

$$\rho' = \mathcal{E}(\rho) = K_0 \rho K_0^\dagger + K_1 \rho K_1^\dagger + K_2 \rho K_2^\dagger$$

$$K_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\lambda_1} & 0 \\ 0 & 0 & \sqrt{1-\lambda_2} \end{pmatrix}, K_1 = \begin{pmatrix} 0 & \sqrt{\lambda_1} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, K_2 = \begin{pmatrix} 0 & 0 & \sqrt{\lambda_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

with $\lambda_m = 1 - e^{-m\Delta t/T_1}$ for gate time Δt

We simulate the errors stochastically, by appending every gate with error operator $\frac{K_i}{\sqrt{p_i}}$ with probability $p_i = \langle\psi|K_i^\dagger K_i|\psi\rangle$.



SIMULATION

We simulated across a range of realistic error rates for near-term superconducting and trapped ion systems.

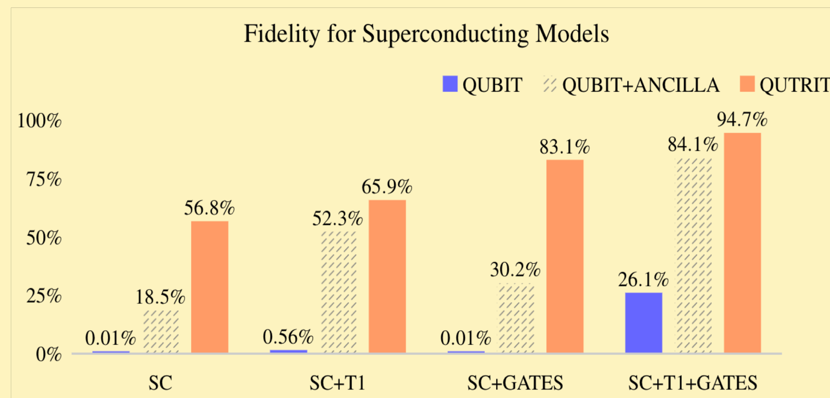
Noise Model	$3p_1$	$15p_2$	T_1
SC	10^{-4}	10^{-3}	1 ms
SC+T1	10^{-4}	10^{-3}	10 ms
SC+GATES	10^{-5}	10^{-4}	1 ms
SC+T1+GATES	10^{-5}	10^{-4}	10 ms

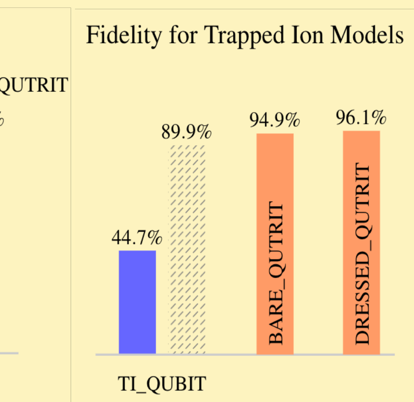
Our SC parameters are based on current IBM hardware as well as experimentally-motivated projections.

Noise Model	p_1	p_2
TI_QUBIT	6.4×10^{-4}	1.3×10^{-4}
BARE_QUTRIT	2.2×10^{-4}	4.3×10^{-4}
DRESSED_QUTRIT	1.5×10^{-4}	3.1×10^{-4}

Our Trapped Ion model is based on dressed qutrit experiments.

We simulated the 13-control Toffoli gate over all error models, using 20,000 CPU hours of simulation time—sufficient to estimate mean fidelities with $2\sigma < 0.1\%$. All error models achieve **>2x** better fidelity for our qutrit design and as high as **10,000x** for current error rates. Due to the asymptotic depth reduction, larger circuits will attain arbitrarily large fidelity advantages.





FUTURE WORK

The primary conclusion of our work is that qutrit states are valuable computational resources. These resources have been traditionally overlooked, but this work demonstrates that they can be used to achieve asymptotically better circuit depths.

Our ongoing work includes:
- a reversible circuit synthesis tool that automatically applies dirty qutrit constructions
- an asymptotically improved qutrit circuit for +k

We also see promising future directions in:
- executing these circuit constructions experimentally to validate the error modeling
- considering other common circuit patterns that could benefit from dirty qutrits constructions.

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