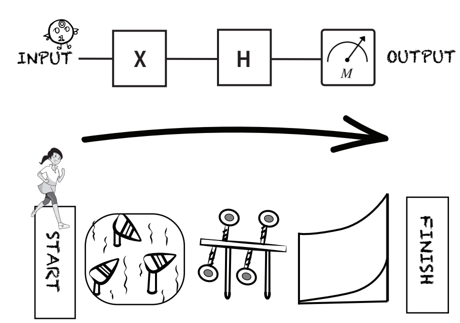
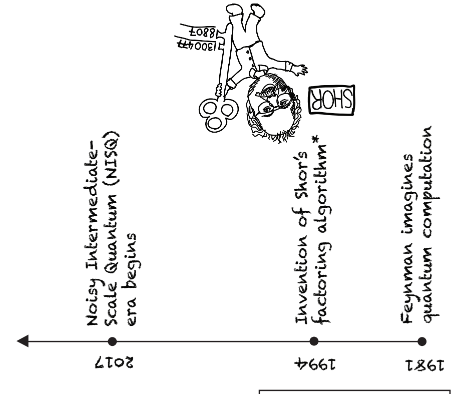
Quantum Circuits



Quantum Circuits

are chronological - like timelines

Time progresses from left to right.

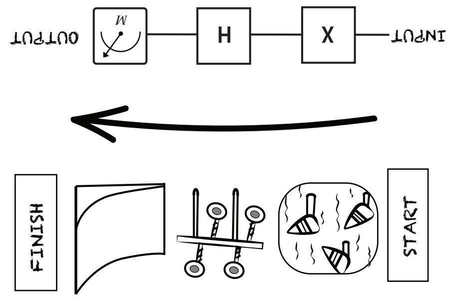


*To learn more about Shor's factoring algorithm, see the EPiQC zine on the history of Quantum Computing.

Quantum Circuits

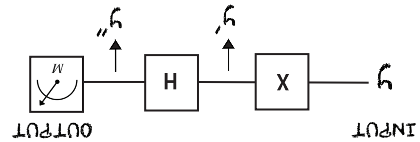
use ordered operations

In American Ninja Warrior, contestants face consecutive obstacles.



Quantum gates operate on qubits, altering their state.

1-Qubit Quantum Circuit



1. First, the X Gate (X) acts on y, producing y'
2. Next, the Hadamard Gate (H) acts on y', producing y''
3. Last, the qubit y'' is measured (M), and the value is 0 or 1

1-Qubit Quantum Circuit Calculations

Note:
$y = |0\rangle = 1|0\rangle + 0|1\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

If $y = |0\rangle$ ...

1: X Gate (X)

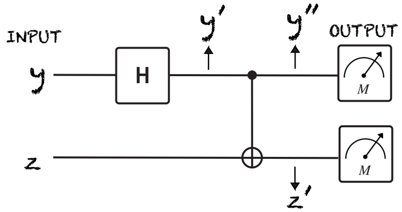
$$\underbrace{\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}}_{\text{Pauli X op}} \underbrace{\begin{bmatrix}1\\0\end{bmatrix}}_{y} = \underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{y'}$$

2: Hadamard Gate (H)

$$\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}}_{\text{H op}} \underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{y'} = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}}_{y''}$$

3: Measurement (M)
y'' has a 50/50 chance of being read as a 0 or a 1

2-Qubit Quantum Circuit



1: First, the Hadamard Gate (H) acts on y, producing y'

2: Next, the C-NOT Gate (⊕) acts on y' and z, producing y'' and z'

3: Last, the qubits y'' and z' are measured, and each qubit's value is a 0 or 1

If $\begin{cases} y = |0\rangle \\ z = |0\rangle \end{cases}$ Let's calculate the result!

2-Qubit Calculations

1. The Hadamard Gate (H) acts on y

$$\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}}_{\text{H}} \underbrace{\begin{bmatrix}1\\0\end{bmatrix}}_{y} = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}}_{y'}$$

2. To apply a 2-qubit gate, first combine the probabilities for y' and z.

$$y' = \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle \text{ and } z = 1|0\rangle + 0|1\rangle$$

$$\tfrac{1}{\sqrt{2}}|00\rangle + 0|01\rangle + \tfrac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

Convert to matrix notation → $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$

3. The C-NOT Gate (⊕) acts on y' and z

$$\underbrace{\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}}_{\text{C-NOT}} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}_{y' \text{ and } z} = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}}_{y'' \text{ and } z'}$$

4. Measurement (M)

There's a 50% chance y''=0 z'=0
0% : y''=0 z'=1
0% : y''=1 z'=0
50% : y''=1 z'=1

Find more Quantum Computing zines here:

https://www.epiqc.cs.uchicago.edu/resources/

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AN NSF EXPEDITION IN COMPUTING EPiQC
https://www.epiqc.cs.uchicago.edu/