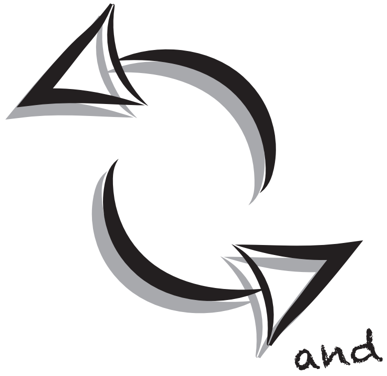
Quantum

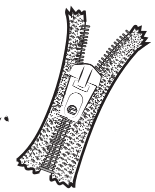


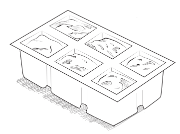
and Reversibility

Reversibility is all around us!

Shoes can be tied & untied.

Zippers open & close with a simple pull.

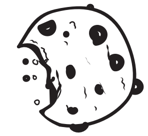


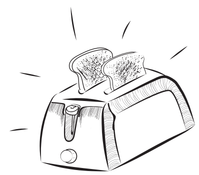


A tray of ice left out to melt refreezes when put back in the freezer.

Some things are NOT reversible

You can't UNBAKE a cookie...





Or UNTOAST bread...

Or UNRING a bell!

Math Operations: SOME are reversible

Negation is reversible.

Given a number: $n = 5$
We can negate the value: $n = -5$
Then reverse the operation: $n = 5$

We return to the original value!

Addition is NOT reversible!

Given only a sum, it's impossible to determine the addends.

For a sum of 8:
$1 + 7 = 8$
$2 + 6 = 8$
$3 + 5 = 8$
$4 + 4 = 8$

$? + ? = 8$

Reversible Addition?

If an addition operation returns one of the input values (x) as part of the output – **Is it reversible?**

$\text{SUM}(x, y) = (x, x + y)$ (inputs → outputs)

When both inputs known, it works like this:

$\text{SUM}(7, 4) = (7, 7 + 4) = (7, 11)$ (inputs → outputs)

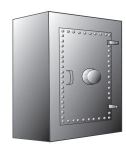
What if y is unknown?
$\text{SUM}(3, y) = (3, 8)$
Can we reverse the operation to find y?

We subtract to find y!
$y = 8 - 3 = 5$
The answer is y = 5!

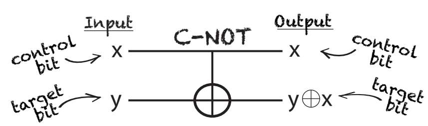
Knowing one of the inputs makes the operation <u>reversible!</u>

This is true of ALL quantum operations. They are reversible because information is preserved!

Quantum Operations MUST be reversible!



Quantum operations are not allowed to lose information.



C-NOT

input x	input y	output x	output y
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

<u>Control bit (x)</u>
NO CHANGE!

<u>Target bit (y)</u>
The control bit (x) determines if the target bit is flipped or stays the same.

Using C-NOT & reversing it!

If we know the input values, the C-NOT truth table can be used to determine the outputs.

$\text{C-NOT}(|0\rangle, |1\rangle) = (|0\rangle, |1\rangle)$ (input → output)

We can also reverse the operation!

If we know the output, we can use the truth table to determine the input.

Now <u>YOU</u> try!

First, go forward:

$\text{C-NOT}(|1\rangle, |0\rangle) = (|\_\_\rangle, |\_\_\rangle)$ (input → output)

Now **reverse** the operation:

$\text{C-NOT}(|\_\_\rangle, |\_\_\rangle) = (|0\rangle, |1\rangle)$ (input → output)

Answer Key:
For inputs $(|1\rangle, |0\rangle)$ – the outputs are $(|1\rangle, |1\rangle)$.
For outputs $(|0\rangle, |1\rangle)$ – the inputs are $(|0\rangle, |1\rangle)$.

For more Quantum Computing Zines visit:

https://www.epiqc.cs.uchicago.edu/resources/

November 2020 (v3)

This work is funded in part by EPiQC, an NSF Expedition in Computing, under grant 1730449

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