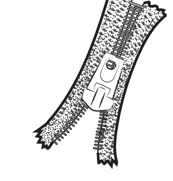
– REVERSIBILITY –

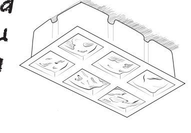
Quantum and Reversibility

Reversibility is all around us!

Shoes can be tied & untied.



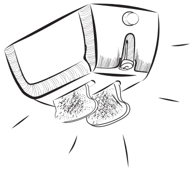
Zippers open & close with a simple pull.



A tray of ice left out to melt refreezes when put back in the freezer.

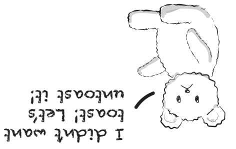
Some things are NOT reversible

You can't UNBAKE a cookie...



Or UNTOAST bread...

Or UNRING a bell!



Math Operations: SOME are reversible

Negation is reversible.

Given a number: n = 5
We can negate the value: n = -5
Then reverse the operation: n = 5

We return to the original value!

Addition is NOT reversible!

Given only a sum, it's impossible to determine the addends.

For a sum of 8:
1 + 7 = 8
2 + 6 = 8
3 + 5 = 8
4 + 4 = 8

? + ? = 8

Reversible Addition?

If an addition operation returns one of the input values (x) as part of the output – **Is it reversible?**

SUM(x, y) = (x, x + y) (inputs → outputs)

When both inputs known, it works like this:

SUM(7, 4) = (7, 7 + 4) = (7, 11) (inputs → outputs)

What if y is unknown?
SUM(3, y) = (3, 8)
Can we reverse the operation to find y?

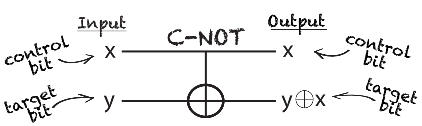
We subtract to find y!
y = 8 - 3 = 5
The answer is y = 5!

Knowing one of the inputs makes the operation **reversible!**

This is true of ALL quantum operations. They are reversible because information is preserved!

Quantum Operations MUST be reversible!

Quantum operations are not allowed to lose information.



C-NOT

input x	input y	output x	output y
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	|1⟩
|1⟩	|1⟩	|1⟩	|0⟩

Control bit (x)
NO CHANGE!

Target bit (y)
The control bit (x) determines if the target bit is flipped or stays the same.

Using C-NOT & reversing it!

If we know the input values, the C-NOT truth table can be used to determine the outputs.

C-NOT(|0⟩, |1⟩)=(|0⟩, |1⟩)
(input → output)

We can also reverse the operation!

If we know the output, we can use the truth table to determine the input.

Now YOU try!

First, go forward:

C-NOT(|1⟩,|0⟩)=(|__⟩,(|__⟩)
(input → output)

Now **reverse** the operation:

C-NOT(|__⟩,|__⟩)=(|0⟩,|1⟩)
(input → output)

Answer Key:
For inputs (|1⟩, |0⟩) – the outputs are (|1⟩, |1⟩).
For outputs (|0⟩, |1⟩) – the inputs are (|0⟩, |1⟩).

For more Quantum Computing Zines visit:

https://www.epiqc.cs.uchicago.edu/resources/

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